# Tabu Search Foundations: Longer Term Memory

# **Frequency-Based Approach**

It provides a type of information that complements the information provided by recency-based memory It broadens the foundation for selecting preferred

moves

Frequencies consist of ratios

Two different measures for numerators:

- *1. transition measure*: the number of iterations where an attribute changes (enters or leaves) the solutions visited on a particular trajectory
- 2. *residence measure*: the number of iterations where an attribute belongs to solutions visited on a particular trajectory

# **Frequency-Based Approach**

Three types for denominators

- 1. Total number of associated iterations represented by the numerator
- 2. The sum (or average) of the numerators
- 3. The maximum numerator value

# **Frequency-Based Approach**

Most applications at present use a simple linear multiple of a frequency measure to create a penalty or incentive term

Problem	Residence Measure	Transition Measure	
Sequencing	Number of times job <i>j</i> has occupied position $\pi(j)$ .	Number of times job <i>i</i> has exchanged positions with job <i>j</i> .	
	Sum of tardiness of job <i>j</i> when this job occupies position $\pi(j)$ .	Number of times job <i>j</i> has been moved to an earlier position in the sequence.	
Min <i>k</i> -Tree Problem	Number of times edge $(i, j)$ has been part of the current solution.	Number of times edge $(i, j)$ has been deleted from the current solution when edge $(k, l)$ has been added.	
	Sum of total solution weight when edge $(i, j)$ is part of the solution.	Number of times edge ( <i>i</i> , <i>j</i> ) has been added during improving moves.	

### **Intensification Strategies**

They initiate a return to attractive (historically found good) regions to search them more thoroughly

Fig. 4.1 Simple TS intensification approach.

Apply TS short term memory. Apply an *elite selection strategy*. **do** { Choose one of the elite solutions. Resume short term memory TS from chosen solution. Add new solutions to elite list when applicable. } while (iterations < limit and list not empty)

# **Intensification Strategies**

Two strategies for selecting elite solutions in Fig. 4.1:

- 1. Introduce a diversification measure to assure the solutions recorded differ from each other by a desired degree
- 2. Keep a bounded length sequential list that adds a new solution at the end only if it is better than any previously seen

- Diversification strategies are designed to derive the search into new regions
- They are often based on modifying choice rules to bring attributes into the solution that are infrequently used

# **Diversification Strategies**

#### Modifying Choice Rules

Graph Partitioning Problem

Full swaps maintain feasibility

- To generate diversity, periodically disallow the use of non-improving full swap
- Partial swaps to allow feasibility after achieving various degrees of infeasibility

Partial swaps with penalty function

*MoveValue*' = *MoveValue* + *d*\**Penalty* 

*Penalty* is often a function of frequency measures *d* is a diversification parameter

Larger *d* corresponds to more diversification

e.g., Nodes that change partitions more frequently are penalized more heavily to encourage the choice of moves that incorporate other nodes

#### Restarting

- Frequency information can be used to design restarting mechanisms
- 6-job sequencing problem
  - Due dates: (9, 12, 15, 8, 20, 22)
  - Initial sequence by EDD rule: (4, 1, 2, 3, 5, 6)

Priority indexes given by the due date values and the frequency information are used for the restarting sequence:

*PriorityIndex'* = *PriorityIndex* +*d\*FrequencyMeasure* 

*FrequencyMeasure* may be the percentage of time that the job was completed on time, considering all the sequences from the last restarting

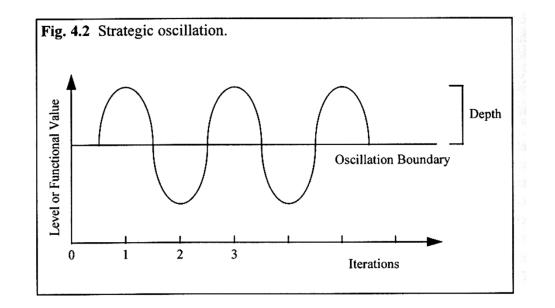
Table 4.2 Illustration of a restarting mechanism.				
Job	PriorityIndex	FrequencyMeasure	PriorityIndex'	
1	9	0.01	9.1	
2	12	0.23	14.3	
3	15	0.83	23.3	
4	8	0.13	9.3	
5	20	0.31	23.1	
6	22	0.93	31.3	
Assume $d = 10$ .				

*Restart* with a new sequence : (1, 4, 2, 5, 3, 6)

A means to achieve an effective interplay between intensification and diversification over the intermediate to long term

- It operates by orienting moves in relation to a critical level, or oscillation boundary
- When this boundary is reached the rules for selecting moves are modified, to permit the region defined by the critical level to be crossed
- The process of repeatedly approaching and crossing the critical level from different directions creates an oscillatory behavior (see Figure 4.2)

The delayed process of finding a route to an optimal solution can be accelerated by means of **logical restructuring** 



#### Min *k*-tree problem:

Edges can be added beyond the critical level defined by k

Then a rule to delete edges must be applied

All feasible solutions lie on the oscillation boundary

Once the oscillation boundary has been reached, the search can stay on it by performing swap moves

The search proceeds to a local optimum each time the critical level is reached

#### **Strategic Oscillation Patterns and Decisions**

Fig. 4.3 Strategic oscillation decisions.

#### **Macro level decisions**

- 1. Select an oscillation guidance function.
- 2. Choose a target level for the function.
- 3. Choose a pattern of oscillation.

#### **Micro level decisions**

- 1. Choose a target rate of change (for moving toward or away from the target level).
- 2. Choose a target band of change.
- 3. Identify aspiration criteria to override target restrictions.

# **Strategic Oscillation**

Macro level decisions:

Oscillation guidance function

e.g., tabu restriction, infeasibility penalties, objective function

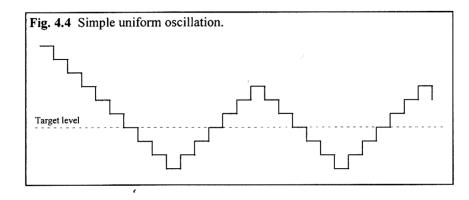
Target level for the function

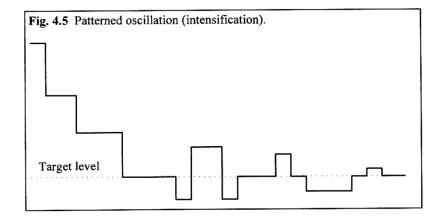
The stage at which current solution constitutes a boundary of feasibility

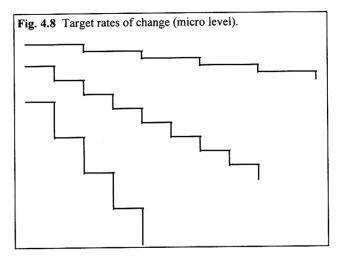
Pattern of Oscillation

The depth by which the search goes beyond the target in a given direction

#### **Strategic Oscillation**







# **Strategic Oscillation**

Micro level decisions: Target rate of change Target band of change Boundaries on deviations from the target rate Aspiration Criteria

# **Path Relinking**

A useful integration of intensification and diversification strategies

- It generates new solutions by exploring trajectories that connect elite solutions
- Starting from one of the elite solution path relinking generates a path in the neighborhood space that leads toward the other solution (*guiding solutions*)
- It may be viewed as an extreme instance of a strategy that seeks to incorporate attributes of high quality solutions

# **Path Relinking**

6-job sequencing problem:

- Processing time: (6, 4, 8, 2, 10, 3)
- Due dates: (9, 12, 15, 8, 20, 22)

Score: the number of jobs that are in the same absolute position in the guiding solution

Improved-best aspiration criterion can allow the path deviate from the goal

### **Path Relinking**

